

3.1 – Vectors in 2-Space, 3-Space, and n -Space

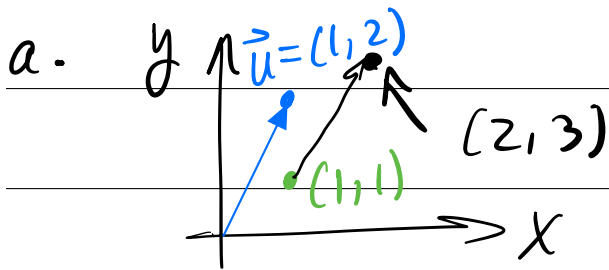
Definition: If n is a positive integer, then an **ordered n -tuple** is a sequence of n real numbers $(v_1, v_2, \dots, v_n)$. The set of all ordered n -tuples is called **real n -space**, denoted by R^n .

R^n

If a vector $\mathbf{v}$ in 2-space or 3-space is positioned with its initial point at the origin of a rectangular coordinate system, then the coordinates of its terminal point are **components** of $\mathbf{v}$ relative to the coordinate system. The numbers in an n -tuple can be considered coordinates of a point or components of a vector.

#5 a. Find the terminal point of the vector that is equivalent to $\mathbf{u} = (1, 2)$ and whose initial point is $A(1, 1)$.

b. Find the initial point of the vector that is equivalent to $\mathbf{u} = (1, 1, 3)$ and whose terminal point is $B(-1, -1, 2)$.



copy of $\vec{u}$:

$$\begin{array}{c} \text{terminal} \quad \text{initial} \\ (2-1, 3-1) \\ = (1, 2) \end{array}$$

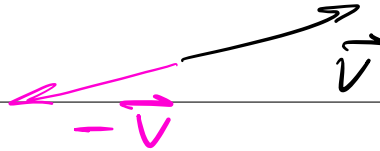
b. $(-1-x_1, -1-y_1, 2-z_1) = (1, 1, 3)$

$$x_1 = -2, x_2 = -2, x_3 = -1$$

The initial point is $(-2, -2, -1)$.

#7 Find the initial point P of a nonzero vector $\mathbf{u} = \overrightarrow{PQ}$ with terminal point $Q(3, 0, -5)$ and such that

- a. $\mathbf{u}$ has the same direction as $\mathbf{v} = (4, -2, -1)$. — Same as previous
b. $\mathbf{u}$ is oppositely directed to $\mathbf{v} = (4, -2, -1)$.



$-\vec{v} = (-4, 2, 1)$, then same as previous

Definition: If $\mathbf{v} = (v_1, v_2, \dots, v_n)$ and $\mathbf{w} = (w_1, w_2, \dots, w_n)$ are vectors in R^n , and if k is any scalar, then we define

- $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n)$ [this is **vector addition**]
- $k\mathbf{v} = (kv_1, kv_2, \dots, kv_n)$ [this is **scalar multiplication**]
- $-\mathbf{v} = (-v_1, -v_2, \dots, -v_n)$
- $\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \dots, w_n - v_n)$

#12 Let $\mathbf{u} = (1, 2, -3, 5, 0)$, $\mathbf{v} = (0, 4, -1, 1, 2)$, and $\mathbf{w} = (7, 1, -4, -2, 3)$.

Find the components of

- a. $\mathbf{v} + \mathbf{w}$
b. $3(2\mathbf{u} - \mathbf{v})$
c. $(3\mathbf{u} - \mathbf{v}) - (2\mathbf{u} + 4\mathbf{w})$
d. $\frac{1}{2}(\mathbf{w} - 5\mathbf{v} + 2\mathbf{u}) + \mathbf{v}$

$$a. \vec{v} + \vec{w} = (0+7, 4+1, -1-4, 1-2, 2+3) \\ = (7, 5, -5, -1, 5)$$

practice b-d.

The **zero vector**, $\vec{0} = (0, 0, \dots, 0)$, has length zero.

$\vec{0}$

Theorem 3.1.1 If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, and if k and m are scalars, then:

- a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) = \mathbf{u} + \mathbf{v} + \mathbf{w}$
- c) $\mathbf{u} + \vec{0} = \vec{0} + \mathbf{u} = \mathbf{u}$
- d) $\mathbf{u} + (-\mathbf{u}) = \vec{0}$
- e) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- f) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$
- g) $k(m\mathbf{u}) = (km)\mathbf{u}$
- h) $1\mathbf{u} = \mathbf{u}$

pf (f): Let $\vec{u} = (u_1, u_2, \dots, u_n)$, $k, m \in \mathbb{R}$.

$$(k+m)\vec{u} = ((k+m)u_1, (k+m)u_2, \dots, (k+m)u_n)$$

$$= (ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n)$$

$$= (ku_1, ku_2, \dots, ku_n) + (mu_1, mu_2, \dots, mu_n)$$

$$= k\vec{u} + m\vec{u}$$

Theorem 3.1.2 If $\mathbf{v}$ is a vector in R^n and k is a scalar, then:

- a) $0\mathbf{v} = \mathbf{0}$
- b) $k\mathbf{0} = \mathbf{0}$
- c) $(-1)\mathbf{v} = -\mathbf{v}$

Definition: if $\mathbf{w}$ is a vector in R^n , then $\mathbf{w}$ is said to be a **linear combination** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r$ in R^n if it can be expressed in the form

$\mathbf{w} = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \dots + k_r\mathbf{v}_r$, where each k_i is a scalar. The scalars are called the **coefficients** of the linear combination. In the case where $r = 1$, we have $\mathbf{w} = k_1\mathbf{v}_1$, in which case the linear combination is a scalar multiple of the vector.

#17 Let $\mathbf{u} = (1, -1, 3, 5)$ and $\mathbf{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\mathbf{u} + b\mathbf{v} = (1, -4, 9, 18)$.

